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PROPAGATION OF SH WAVES IN AN REGULAR NON HOMOGENEOUS MONOCLINIC CRUSTAL LAYER LYING OVER A NON-HOMOGENEOUS SEMI-INFINITE MEDIUM

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The present paper discusses the dispersion equation for SH waves in a non-homogeneous monoclinic layer over a semi infinite isotropic medium. The wave velocity equation has been obtained. In the isotropic case, when non-homogeneity is absent, the dispersion equation reduces to the standard SH wave equation. The dispersion curves are depicted by means of graphs for different values of non-homogeneity parameters for the layer and semi-infinite medium.

Key words: SH waves, monoclinic, non homogeneity, differential equations.

1. Introduction

The formulations and solutions of many problems of linear wave-propagation for homogeneous media are available in the literature of continuum mechanics of solids. In recent years, however, considerable interest has arisen in the problem connected with bodies whose mechanical properties are functions of space, i.e. non-homogeneous bodies. This interest is mainly due to the advent of solid rocket propellants, polymeric materials and growing demand of engineering and industrial applications.

The propagation of surface waves in elastic media is of considerable importance in earth-quake engineering and seismology on account of the occurrence of stratification in the earth crust, as the earth is made up of different layers. As a result, the theory of surface waves was developed by Stoneley [1], Bullen [2], Ewing *et al*. [3], Hunters [4] and Jeffreys [5].

Many results of theoretical and experimental studies revealed that the real earth is considerably more complicated than the models presented earlier. This has led to a need for more realistic representation as a medium through which seismic waves propagate. The wave propagation in crystalline media plays a very interesting role in geophysics and also in ultrasonic and signal processing. A monoclinic medium is an example of such medium, keeping in mind the fact that the non-homogeneity characteristic is one of the most generalized elastic conditions inside the earth. Many authors have studied the propagation of different waves in different media with non-homogeneity.

Sezawa [6] studied the dispersion of elastic waves propagated on curved surfaces. The transmission of elastic waves through a stratified solid medium was studied by Thomson [7]. Haskell [8] studied the dispersion of surface waves in multilayered media. Biot [9] studied the influence of gravity on Rayleigh

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waves, assuming the force of gravity to create a type of initial stress of hydrostatic nature and the medium to be incompressible.

Propagation of Love waves in a non-homogeneous stratum of finite depth sandwiched between two semi infinite isotropic media was studied earlier by Sinha [10]. Wave propagation in a thin two-layered laminated medium with couple under initial stress was studied by Roy [11]. Datta [12] studied the effect of gravity on Rayleigh wave propagation in a homogeneous, isotropic elastic solid medium. Effects of irregularities on the propagation of guided SH waves were studied by Chattopadhyay *et al.* [13]. Goda [14] studied the effect of non-homogeneity and anisotropy on Stoneley waves. Gupta *et al*. [15] investigated the influence of linearly varying density and rigidity on torsional surface waves in a non-homogeneous crustal layer.

Some of the recent notable works on the propagation of seismic waves in various media with different geometries were written by Chattopadhyay *et al*. [16-18].

Recently Sethi and Gupta [19] investigated the surface waves in homogeneous viscoelastic media of higher order under the influence of surface stresses.

In the present problem, we have considered the propagation of SH wave in a regular monoclinic crustal layer over an isotropic semi-infinite medium. The dispersion relation is found in the closed form and matched with the classical Love wave equation as a particular case. The dispersion curves are depicted by means of graphs for different values of non-homogeneity parameters. The influence of non-homogeneity parameters, wave number and the thickness of the layer on the dimensionless phase velocity has been studied.

2. Formulation of the problem

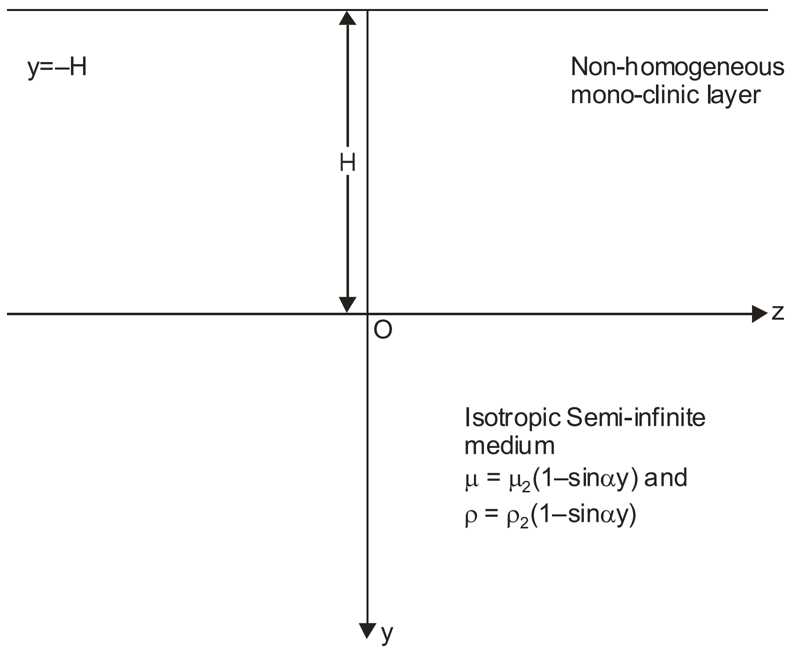

Fig1. Geometry of the problem.

Let us consider ρ_i, u_i ($i = 1, 2$) as the densities and displacements in a monoclinic layer (of thickness H) and a semi-infinite isotropic medium, respectively.

Assuming the z-axis along the interface of the layer and semi-infinite medium, the y-axis is taken vertically downwards.

First, we will deduce the equation of motion for propagation of SH wave in a monoclinic layer,

For a monoclinic layer, we have the following strain-displacement relation

$$S_1 = \frac{\partial u}{\partial x}, \quad S_2 = \frac{\partial v}{\partial y}, \quad S_3 = \frac{\partial w}{\partial z}, \quad S_4 = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}, \quad S_5 = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}, \quad S_6 = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \tag{2.1}$$

where u, v, w are displacements along the x, y, z axis respectively and S_i ($i=1, 2, \ldots\ldots, 6$) are strain components.

The stress-strain relations for a rotated y-cut quartz plate, which exhibits monoclinic symmetry with x being the diagonal axis, are

$$\begin{aligned}
T_1 &= C_{11}S_1 + C_{12}S_2 + C_{13}S_3 + C_{14}S_4, \\
T_2 &= C_{12}S_1 + C_{22}S_2 + C_{13}S_3 + C_{14}S_4, \\
T_3 &= C_{13}S_1 + C_{23}S_2 + C_{33}S_3 + C_{34}S_4, \\
T_4 &= C_{14}S_1 + C_{24}S_2 + C_{34}S_3 + C_{44}S_4, \\
T_5 &= C_{55}S_5 + C_{56}S_6, \\
T_6 &= C_{56}S_5 + C_{66}S_6
\end{aligned} \tag{2.2}$$

where T_i ($i= 1, 2, \ldots\ldots, 6$) are stress components and $C_{ij}= C_{ji}$ ($i, j = 1, 2, \ldots\ldots\ldots, 6$) are medium (elastic) constants.

The equation of motion in the absence of body forces are

$$\begin{aligned}
\frac{\partial T_1}{\partial x} + \frac{\partial T_6}{\partial y} + \frac{\partial T_5}{\partial z} &= \rho \frac{\partial^2 u}{\partial t^2}, \\
\frac{\partial T_6}{\partial x} + \frac{\partial T_2}{\partial y} + \frac{\partial T_4}{\partial z} &= \rho \frac{\partial^2 v}{\partial t^2},
\end{aligned} \tag{2.3}$$

and

$$\frac{\partial T_5}{\partial x} + \frac{\partial T_4}{\partial y} + \frac{\partial T_3}{\partial z} = \rho \frac{\partial^2 w}{\partial t^2}$$

where ρ is the density of the upper monoclinic layer.

For SH waves propagating in the z-direction with the displacement only in the x-direction, we have

$$u = u(y, z, t), \quad v = 0, \quad w = 0. \tag{2.4}$$

Using Eqs (2.1) and (2.4), we get

$$S_1 = 0, \quad S_2 = 0, \quad S_3 = 0, \quad S_4 = 0, \quad S_5 = \frac{\partial u}{\partial z}, \quad S_6 = \frac{\partial u}{\partial y}. \tag{2.5}$$

With the help of Eqs (2.5) Eqs (2.2) gives

$$T_1 = T_2 = T_3 = T_4 = 0, \quad T_5 = C_{55}\frac{\partial u}{\partial z} + C_{56}\frac{\partial u}{\partial y} \quad \text{and} \quad T_6 = C_{56}\frac{\partial u}{\partial z} + C_{66}\frac{\partial u}{\partial y}. \tag{2.6}$$

3. Solution for monoclinic layer

Let the non-homogeneities for the monoclinic layer be considered as

$$C_{66} = C'_{66}e^{my}, \quad C_{56} = C'_{56}e^{my}, \quad C_{55} = C'_{55}e^{my}, \quad \rho = \rho_1 e^{my}. \tag{3.1}$$

Using Eqs (2.4), (2.6) and (3.1) in Eq.(2.3), we obtain the non-vanishing equation of motion as

$$C'_{66}\frac{\partial^2 u_1}{\partial y^2} + 2C'_{56}\frac{\partial^2 u_1}{\partial y \partial z} + C'_{55}\frac{\partial^2 u_1}{\partial z^2} + mC'_{56}\frac{\partial u_1}{\partial z} + mC'_{66}\frac{\partial u_1}{\partial y} = \rho_1\frac{\partial^2 u_1}{\partial t^2}. \tag{3.2}$$

We can assume that the solution of Eq.(3.2) is

$$u_1(y,z,t) = U_1(y)e^{iK(z-ct)} \tag{3.3}$$

where K is the wave number and c is the velocity of SH wave.

$$\frac{d^2U_1}{dy^2} + \left(2iK\frac{C'_{56}}{C'_{66}} + m\right)\frac{dU_1}{dy} + \left[\frac{C'_{55}\left(-k^2\right) + ikmC'_{56} + \rho_1\omega^2}{C'_{66}}\right]U_1 = 0. \tag{3.4}$$

By introducing $U_1 = V(y)e^{-a_1 y/2}$ where $a_1 = \left(2iK\frac{C'_{56}}{C'_{66}} + m\right)$ in Eq.(3.4), we have

$$\frac{d^2V}{dy^2} + \left[\frac{-a_1^2}{4} - \frac{C'_{55}}{C'_{66}}K^2 + iKm\frac{C'_{56}}{C'_{66}} + \frac{\rho_1\omega^2}{C'_{66}}\right]V = 0. \tag{3.5}$$

The solution of Eq.(3.5) can be taken as

$$V(y) = \left(A\cos Ty + B\sin Ty\right)$$

where

$$T^2 = K^2\left[-\frac{m^2}{4K^2} + \left(\frac{C'_{56}}{C'_{66}}\right)^2 - \frac{C'_{55}}{C'_{66}} + \frac{c^2}{\beta_1^2}\right]$$

where

$$\beta_1^2 = \frac{C'_{66}}{\rho_1}.$$

Thus solution of the layer becomes

$$u_1(y,z,t)=\left[A\cos Ty+B\sin Ty\right]e^{-a_1 y/2}e^{i(Kz-\omega t)}. \tag{3.6}$$

4. Solution for semi infinite half space

For Love wave propagation, we have

$$u=w=0 \qquad \text{and} \qquad v=v\,(y,z,t\,). \tag{4.1}$$

The equations governing the propagation of Love wave in a homogeneous isotropic elastic medium in the absence of body forces are

$$\begin{aligned}
&\frac{\partial}{\partial x}\tau_{xx}+\frac{\partial}{\partial y}\tau_{yx}+\frac{\partial}{\partial z}\tau_{zx}=\rho\frac{\partial^2 u}{\partial t^2},\\
&\frac{\partial}{\partial x}\tau_{xy}+\frac{\partial}{\partial y}\tau_{yy}+\frac{\partial}{\partial z}\tau_{zy}=\rho\frac{\partial^2 v}{\partial t^2},\\
&\frac{\partial}{\partial x}\tau_{xz}+\frac{\partial}{\partial y}\tau_{yz}+\frac{\partial}{\partial z}\tau_{zz}=\rho\frac{\partial^2 w}{\partial t^2}.
\end{aligned} \tag{4.2}$$

Also, Hooke's law for isotropic medium,

$$\tau_{ij}=\lambda\Delta\delta_{ij}+2\mu\varepsilon_{ij} \tag{4.3}$$

where λ, μ are Lame's constants and Δ is the dilatation.

Also $$\varepsilon_{ij}=\frac{1}{2}\left[\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right]. \tag{4.4}$$

In view of Eqs (2.4), (4.3) and (4.4), the equation of motion (4.2) gives the non-vanishing equations of motion for propagation of SH wave in the lower semi-infinite medium as

$$\frac{\partial}{\partial y}\left(\mu\frac{\partial u_2}{\partial y}\right)+\frac{\partial}{\partial z}\left(\mu\frac{\partial u_2}{\partial z}\right)=\rho\frac{\partial^2 u_2}{\partial t}. \tag{4.5}$$

For wave propogating in the *z*-direction

$$u_2=W(y)e^{i(kz-wt)}, \tag{4.6}$$

Eq.(4.5) reduces to

$$\frac{d^2W}{dy^2}+\frac{1}{\mu}\frac{d\mu}{dy}\frac{dW}{dy}+K^2\left(\frac{\rho c^2}{\mu}-1\right)W=0\,. \tag{4.7}$$

Introducing $W=\frac{w_1}{\sqrt{\mu}}$ in Eq.(4.7), to eliminate $\frac{dW}{dy}$, we get

$$\frac{d^2W_1}{dy^2}-\frac{1}{2\mu}\frac{d^2\mu}{dy^2}W_1+\frac{1}{4\mu^2}\left(\frac{d\mu}{dy}\right)^2 W_1+k^2\left(\frac{\rho c^2}{\mu}-1\right)W_1=0\,. \tag{4.8}$$

We take variations in rigidity and density as

$$\mu=\mu_2\left(1-\sin\alpha y\right);\quad \rho=\rho_2\left(1-\sin\alpha y\right),\quad \alpha>0\,. \tag{4.9}$$

Introducing Eqs (4.9) in Eq.(4.8), we get

$$\frac{d^2W_1}{dy^2}-T_1^2W_1=0;$$

where $$T_1^2=K^2\left(1-\frac{c^2}{\beta_2^2}-\frac{\alpha^2}{4K^2}\right),\quad \beta_2=\sqrt{\frac{\mu_2}{\rho_2}}\,. \tag{4.10}$$

Thus the solution for Eqs (4.10) can be taken as

$$W_1=e^{T_1y}+e^{-T_1y}\,.$$

Thus the displacement component for a non-homogenous half space is given by

$$u_2\left(y,z,t\right)=\frac{Ce^{-T_1y}}{\left(1-\sin\alpha y\right)^{1/2}}e^{i\left(Kz-wt\right)}\,. \tag{4.11}$$

5. Boundary conditions

The boundary conditions are as follows:

(i) The upper monoclinic layer is stress free, i.e., $T_6=0$, at $y=-H$

$$C_{56}\frac{\partial u_1}{\partial z}+C_{66}\frac{\partial u_1}{\partial y}=0 \quad \text{at} \quad y=-H. \tag{5.1}$$

(ii) The stresses are continuous at a common interface

$$C_{56}\frac{\partial u_1}{\partial z}+C_{66}\frac{\partial u_1}{\partial y}=\mu_2\frac{\partial u_2}{\partial y} \quad \text{at} \quad y=0. \tag{5.2}$$

(iii) The displacements are continuous at a common interface

$$u_1 = u_2 \quad \text{at} \quad y = 0. \tag{5.3}$$

Using all the boundary conditions (5.1), (5.2), (5.3) and after simplification, we have

$$A\left[C'_{66}T\sin TH + \left(C'_{56}ik - \frac{a_1}{2}C'_{66}\right)\cos TH\right] + B\left[\left(-ikC'_{56} + \frac{a_1}{2}C'_{66}\right)\sin TH + C'_{66}T\cos TH\right] = 0 \tag{5.4}$$

$$A\left[C'_{56}ik - \frac{a_1}{2}C'_{66}\right] + BC'_{66}T = -\mu_2\left(T_1 - \frac{\alpha}{2}\right)C, \tag{5.5}$$

$$A = C. \tag{5.6}$$

Eliminating A, B, C from Eqs (5.4), (5.5) and (5.6), we get

$$\text{Det}\,(D_{ij}) = 0, \quad \text{where} \quad i, j = 1, 2, 3 \tag{5.7}$$

where

$$D_{11} = C'_{66}T\sin TH - \frac{m}{2}C'_{56}\cos TH\,; \quad D_{12} = \frac{m}{2}C'_{56}\sin TH + C'_{66}T\cos TH; \quad D_{13} = 0,$$

$$D_{21} = \frac{-m}{2}C'_{56}; \quad D_{22} = C'_{66}T; \quad D_{23} = \left(T_1 - \frac{\alpha}{2}\right)\mu_2\,;$$

$$D_{31} = 1; \quad D_{32} = 0; \quad D_{33} = -1\ .$$

After simplification of Eq.(5.7), we have

$$\tan(TH) = \frac{A_1}{A_2} \tag{5.8}$$

where

$$A_1 = T\left(T_1 - \frac{\alpha}{2}\right)\frac{\mu_2}{C'_{66}},$$

$$A_2 = T^2 + \frac{m^2}{4} - \left(T_1 - \frac{\alpha}{2}\right)\frac{m}{2}\frac{\mu_2}{C'_{66}}.$$

After substituting the values of T and T_1 in Eq.(5.8), we get

$$\tan\left(KH\sqrt{-\left(\frac{m}{2k}\right)^2+\left[\left(\frac{C'_{56}}{C'_{66}}\right)^2-\frac{C'_{55}}{C'_{66}}\right]+\frac{c^2}{\beta_1^2}}\right)=\frac{A_1}{A_2} \tag{5.9}$$

where

$$A_1=\frac{\mu_2}{C'_{66}}\left[\sqrt{1-\frac{c^2}{\beta_2^2}-\left(\frac{\alpha}{2K}\right)^2}-\frac{\alpha}{2K}\right]\sqrt{-\left(\frac{m}{2K}\right)^2+\left[\left(\frac{C'_{56}}{C'_{66}}\right)^2-\frac{C'_{55}}{C'_{66}}\right]+\frac{c^2}{\beta_1^2}}\,,$$

$$A_2=\left(\frac{C'_{56}}{C'_{66}}\right)^2-\frac{C'_{55}}{C'_{66}}+\frac{c^2}{\beta_1^2}-\frac{\mu_2}{C'_{66}}\frac{m}{2K}\left[\sqrt{1-\frac{c^2}{\beta_2^2}-\left(\frac{\alpha}{2K}\right)^2}-\frac{\alpha}{2K}\right],$$

which gives the dispersion relation for propagation of SH waves in a non-homogeneous monoclinic layer lying over an isotropic non-homogeneous semi-infinite medium.

6. Particular cases

Case (I): When $C'_{66} = C'_{55}=\mu_1,\ C'_{56}=0$, Eq.(5.9) reduces to,

$$\tan\left(KH\sqrt{-1+\frac{c^2}{\beta_1^2}}\right)=\frac{A_3}{A_4} \tag{6.1}$$

where

$$A_3=\frac{\mu_2}{\mu_1}\left(\sqrt{-1+\frac{c^2}{\beta_2^2}-\left(\frac{\alpha}{2K}\right)^2}-\frac{\alpha}{2K}\right),$$

$$A_4=\sqrt{-1+\frac{c^2}{\beta_1^2}}\,,$$

which gives the wave velocity equation for propagation of SH waves in a non-homogeneous isotropic layer lying over an isotropic non-homogeneous semi-infinite medium.

Case (II): When $m=0$, $C'_{66} = C'_{55}=\mu_1,\ C'_{56}=0$, Eq.(5.9) reduces to

$$\tan\left(KH\sqrt{\frac{c^2}{\beta_1^2}-1}\right)=\frac{A_5}{A_6} \tag{6.2}$$

where

$$A_5=\frac{\mu_2}{\mu_1}\left(\sqrt{-1+\frac{c^2}{\beta_2^2}-\left(\frac{\alpha}{2K}\right)^2}-\frac{\alpha}{2K}\right),$$

$$A_6=\sqrt{\frac{c^2}{\beta_1^2}-1}\,,$$

which gives the dispersion relation for propagation of SH waves in a homogeneous isotropic layer lying over a non-homogeneous isotropic semi-infinite medium.

Case (III): When $m=0$, $\alpha=0$, $C'_{66} = C'_{55}=\mu_1$, $C'_{56}=0$, Eq.(5.9) reduces to

$$\tan\left(KH\sqrt{\frac{c^2}{\beta_1^2}-1}\right)=\frac{A_7}{A_8} \tag{6.3}$$

where

$$A_7=\frac{\mu_2}{\mu_1}\left(\sqrt{1-\frac{c^2}{\beta_2^2}}\right),$$

$$A_8=\sqrt{\frac{c^2}{\beta_1^2}-1},$$

which is the dispersion relation for propagation of SH waves in a homogeneous isotropic layer lying over a homogeneous isotropic semi-infinite medium, which is in complete agreement with the classical Love wave equation.

Case (IV): When $m=0$, Eq.(5.9) reduces to

$$\tan\left(KH\sqrt{\frac{c^2}{\beta_1^2}+\left(\frac{C'_{56}}{C'_{66}}\right)^2-\frac{C'_{55}}{C'_{66}}}\right)=\frac{A_9}{A_{10}} \tag{6.4}$$

where

$$A_9=\frac{\mu_2}{C'_{66}}\left(\sqrt{1-\frac{c^2}{\beta_2^2}-\left(\frac{\alpha}{2K}\right)^2}-\frac{\alpha}{2K}\right),$$

$$A_{10}=\sqrt{\left[\left(\frac{C'_{56}}{C'_{66}}\right)^2-\frac{C'_{55}}{C'_{66}}\right]+\frac{c^2}{\beta_1^2}},$$

which gives the dispersion relation for propagation of SH waves in a homogeneous monoclinic layer lying over a non-homogeneous isotropic semi-infinite medium.

Case (V): When $m=0$, $\alpha=0$ Eq.(5.9) reduces to

$$\tan\left(KH\sqrt{\frac{c^2}{\beta_1^2}+\left(\frac{C'_{56}}{C'_{66}}\right)^2-\frac{C'_{55}}{C'_{66}}}\right)=\frac{A_{11}}{A_{12}} \tag{6.5}$$

where

$$A_{11}=\frac{\mu_2}{C'_{66}}\left[\sqrt{1-\frac{c^2}{\beta_2^2}}\right],$$

$$A_{12} = \sqrt{\left[\left(\frac{C'_{56}}{C'_{66}}\right)^2 - \frac{C'_{55}}{C'_{66}}\right] + \frac{c^2}{\beta_1^2}}\ ,$$

which gives the dispersion relation for propagation of SH waves in a homogeneous monoclinic layer lying over an isotropic homogeneous semi-infinite medium, which is in complete in agreement with the corresponding classical result given by Chattopadhyay and Pal.

7. Numerical computations and discussion

To study the effect of various dispersion non-homogeneities on the propagation of SH wave propagating in non-homogeneous mono-clinic layer lying over a non-homogeneous semi-infinite media, phase velocity is calculated numerically with the help of MATLAB for Eq.(5.9). We take the following data: for a mono-clinic layer (Tiersten [22])

$$C'_{55} = 94\times 10^9\ N/m^2\ , \qquad C'_{56} = -11\times 10^9\ N/m^2\ ,$$

$$C'_{66} = 93\times 10^9\ N/m^2\ , \qquad \rho_1 = 7450\,Kg/m^3\ .$$

for a semi-infinite medium (Gubbins [21])

$$\mu_2 = 6.54\times 10^{10}\ N/m^2\ , \qquad \rho_2 = 3409\,Kg/m^3\ .$$

The effect of exponentially varying elastic parameters and density on SH waves in a non-homogeneous monoclinic crustal layer over a non-homogeneous half space is discussed in the following way by means of graphs.

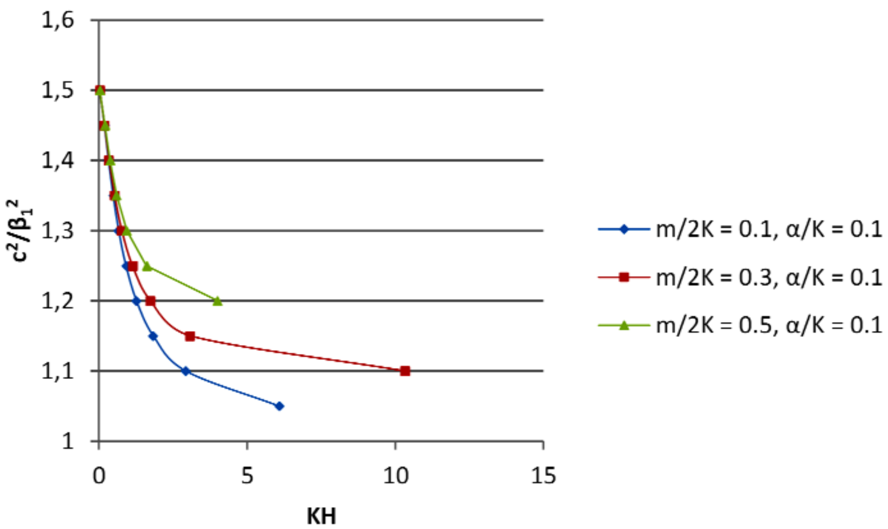

Fig.2. Variation of dimensionless phase velocity $(c/\beta_1)^2$ against dimensionless wave number KH demonstrating the influence of non-homogeneity associated with mono-clinic crustal layer.

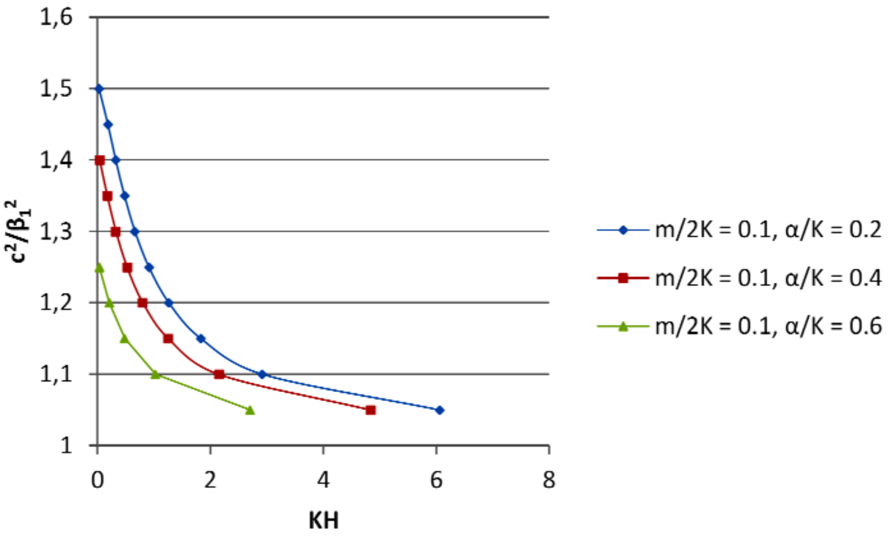

Fig.3. Variation of dimensionless phase velocity $(c/\beta_1)^2$ against dimensionless wave number KH demonstrating the influence of non-homogeneity associated with half-space.

Figure 2 shows the effect of non-homogeneity parameter $m/2K$ involved in the rigidity of the monoclinic crustal layer when a non-homogeneous half space (i.e., rigidity and density varying trigonometrically with depth) is taken into consideration. The following observations are mode and effects obtained under the above considered values.

(1a) For a particular dimensionless wave number KH and a fixed value of non-homogeneity of the half space i.e., $\alpha/K = 0.2$, the dimensionless phase velocity $(c/\beta_1)^2$ of SH waves increases as the value of $m/2K$ increases from 0.1 to 0.5.

(1b) For various values of $m/2K$ and a fixed value of α/K, the phase velocity $(c/\beta_1)^2$ increases as the wave number decreases in all curves 1-3.

(1c) Curve 1(for $m/2K=0.5$) is steeper than curve 2 (for $m/2K=0.3$), which in turn is steeper than curve 3(for $m/2K=0.1$), which reveals that the dimensionless non-homogeneity factor $m/2K$ has a prominent effect on SH wave propagation.

(1d) All the three curves are coinciding as the wave number approaches 0.1.

Figure 3 shows the effect of non-homogeneity parameter α/K involved in the rigidity and density of the non-homogeneous half space where a non-homogeneous monoclinic crustal layer lying over it whose (i.e., elastic parameters and density vary exponentially with depth) is taken into consideration. The following observations are mode and effects obtained under the above considered values.

(2a) For a particular dimensionless wave number KH and a fixed value of non-homogeneity of the layer i.e., $m/2K = 0.1$, the dimensionless phase velocity $(c/\beta_1)^2$ of SH waves decreases as the value of α/K increases from 0.2 to 0.6.

(2b) For various values of α/K and a fixed value of $m/2K$, the phase velocity increases as the wave number decreases in all curves 1-3.

(2c) Curve 1(for $\alpha/K=0.6$) is steeper than curve 2 (for $\alpha/K=0.4$), which in turn is steeper than curve 3(for $\alpha/K=0.2$), which reveals that the dimensionless non-homogeneity factor α/K has a prominent effect on SH wave propagation.

8. Conclusions

Here, we studied the propagation of the SH waves in a non-homogeneous mono-clinic crustal layer lying over a non-homogeneous semi-infinite media. A closed form solutions were derived separately for the displacements in monoclinic layer and half-space. By using the asymptotic expansion of Whittaker's function we derived the wave velocity equation for the SH waves in compact form. Dimensionless phase velocity is calculated numerically with the help of MATLAB. The effect of various dimensionless elastic parameters and non-homogeneity factors on the dimensionless phase velocity $(c/\beta_1)^2$ have been shown graphically. We make the following observations

1. For various values of $m/2K$ and a fixed value of α/K, the phase velocity $(c/\beta_1)^2$ increases as the wave number decreases.
2. For a particular dimensionless wave number KH and a fixed value of non-homogeneity of the half space i.e., α/K, the dimensionless phase velocity $(c/\beta_1)^2$ of SH waves increases as the value of $m/2K$ increases.
3. For a particular dimensionless wave number KH and a fixed value of non-homogeneity of the layer i.e., $m/2K$, the dimensionless phase velocity $(c/\beta_1)^2$ of SH waves increases as the value of α/K increases.
4. In the absence of all non-homogeneities in the density and rigidity of a monoclinic layer and semi-infinite half-space, the dispersion equation for the propagation of SH waves in a homogeneous monoclinic layer lying over an isotropic homogeneous semi-infinite medium is in complete agreement with the classical dispersion equation of Chattopadhyay and Pal.
5. In the absence of all non-homogeneities in the density and rigidity and $C'_{66} = C'_{55}=\mu_1$, $C'_{56} =0$, the dispersion equation for the propagation of SH waves in an isotropic homogeneous layer lying over an isotropic homogeneous semi-infinite medium is in complete agreement with the classical dispersion equation of Love wave.

The wave propagation in crystalline media (monoclinic media) plays a very interesting role in geophysics and also in ultrasonic and signal processing. This study is helpful in understanding the cause and estimating of damage due to earthquakes. The present paper may be useful in predicting the behavior of SH waves in non-homogeneous crystalline geo-media.

Nomenclature

$C_{ij}, i, j = 1, 2 ......, 6$ – elastic constants
c – velocity of SH waves
H – thickness of the layer
K – wave number
m – non-homogeneity parameter in the monoclinic layer
$S_i, i = 1$ to 6 – strain components
$T_i, i = 1$ to 6 – stress components
α – non-homogeneity in the semi-infinite medium
Δ – dilatation
λ, μ – Lame’s constants
$\rho_i, i = 1$ to 2 – densities of the monoclinic layer and semi-infinite medium

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